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Black-Scholes Model in Context

Interview Robert C. Merton

Origins of the Black-Scholes Model

Special

INTERVIEW

The Black-Scholes Model in Context

The Black-Scholes option pricing model, first published by Black and Scholes in 1973, has had a major impact on financial markets and option pricing. In 1997 Myron S. Scholes and Robert C. Merton were awarded the Nobel Prize in economics "for a new method to value derivatives". Fischer Black would have shared the prize if he had still been alive at the time.

Name:
Robert C. Merton

Position:
Distinguished Professor of Finance

Nekst has had an interview with Robert C. Merton, MIT Sloan School of Management Distinguished Professor of Finance, about the time that he worked on the proof of what is today known as the Black-Scholes model. Professor Schumacher kindly wrote an article about the origins of the Black-Scholes formula which can be found on pages 36-39. This article can be considered as background knowledge which can be useful when reading the interview with Robert Merton below.

During the time that Fischer Black and Myron Scholes were working on their formula you started working in the same field. Later you found out what Black and Scholes were doing. Intense discussions on the model followed as Black and Scholes based their solution on the Capital Asset Pricing Model. How would you describe this time working with Black and Scholes and how did these discussions develop?

"Well as a bit of background, I was a graduate student at MIT and had basically lived in Paul Samuelson's office as his research assistant. In my first year he hired me and we did research together. Our first research paper was on the pricing of warrants, based on his earlier papers. I had been involved in trading of warrants, convertible bonds, and options in real-world markets. The other research problem I started to work on at that time which is relevant to where things went was lifetime consumption-portfolio optimization, how over a person's lifetime they would optimally consume and allocate their investments across risky assets and the risk-free asset through a lifetime in an uncertain world. I developed and published models on that problem and to do that I discovered a mathematical tool, Itô calculus, for modeling the actual sample paths that portfolios might follow, not just their expectations but their realizations. I used it to build models of a dynamic portfolio optimization. So that is the background.

As I said, I was working on warrants and options, which we would now call derivatives, first with Paul and then on my own. In 1969 when I was finishing my degree to take a job in 1970, Franco Modigliani suggested to me that perhaps I would like to stay at MIT on the faculty at the Sloan School of Management. I found it very interesting since my research was going so well at MIT. So I went to see who the other Professors were and that is when I met Myron Scholes who was a young assistant professor from the University of Chicago. When I joined the Sloan faculty in the summer of 1970 I had discussions with him about what he was working on. He was mostly pursuing empirical work, but he also had this project with Fischer Black on option pricing. Gradually through time he told me what they were trying to do and the idea of employing a dynamic trading strategy to create a hedge portfolio to hedge out the risk of an option using the underlying stock and cash. Initially I was skeptical that they could get pricing to work this way, because options are nonlinear functions of the stock price. But then I put it into the context of the continuous-time, continuous-trading model that I had been using to model optimal dynamic portfolio strategies. When I tried their proposed hedging strategy in the context of my continuous-time technology I found out that it not only worked, in fact, it worked even better than they had thought.

My contribution relative to theirs was that they had the idea of using a dynamic strategy to hedge the systematic (aka beta) risk of the option and, thereby derive a pricing formula based on the equilibrium CAPM. I was able to show, provided you could trade very frequently, that you could essentially create a zero-total-risk portfolio that did not depend on any equilibrium model, but it depended on the much weaker condition of no arbitrage. Furthermore, since you can get rid of all the risk, you must have a portfolio strategy that exactly replicates the pay-off of the option.

Fischer Black and Myron Scholes worked together and I worked by myself. Of course Myron Scholes and I were in the same department and we talked, but we never collaborated in the sense of sitting around a table with the three of us and having conversations on the modelling. It was more that we were working in parallel and it was Myron Scholes whom I talked with. It was a very magical time for doing finance research. This breakthrough opened the door to a vast cornucopia of research problems which could be solved with the replicating portfolio approach. I had the great good fortune to be at its beginning and to help develop some of this technology. There were new research papers to be written every day."

During the period working on the Black-Scholes model, what did an average day look like?
"I lived in an MIT apartment building 50 meters from my office and so I did all my work in the office. I worked on my research and I did teaching and wrote teaching notes. Some might call it work, but it was fun, it was exciting. You are trying to find the answers to puzzles and challenges. I would never get it out of my mind because I was always thinking there were more exciting things to discover. There were more interesting problems to work on than there was time to do so. I was not a workaholic; it was more that this was exciting and interesting. Who would not want to do it?"

We have read that Black recalled: "Though the search for the formula was an academic search for truth, we did try to use it to make money." Was this a real motive to work on the model, or was it simply a sidetrack?
"The direct answer is that making money was just a sidetrack. That said, I had always been involved in the market. I bought my first share of stock when I was ten years old and I used to trade the stock, OTC option, and convertible markets when I was a student at California Institute of Technology before I came to MIT. Markets opened at 9:30h in New York which was 6:30h in California, so I could trade them a couple of hours before going to classes. So I was very much involved in practice before I even studied economics. It was true that I did trade, but the prime motivation for working on the option-pricing problem was not to make money. It was an intellectually challenging problem, it was fun and interesting. For instance, it never occurred to us to keep our finding a secret and not put it in the public domain and publish it. Fischer Black's remark was probably referring to one specific warrant trade that we did, which had an object lesson in it."

You are responsible for labeling the Black-Scholes model for the first time as to what it is called today. Black said on the paper he published with Scholes that "It should probably be called the Black-Merton-Scholes paper." After helping them in solving their problem of derivation, did you ever talk about this name with either Black or Scholes and to what degree do you see it as their intellectual property?
"The answer is, no, I never talked to them about it. First of all, I named it. It would be rather pretentious to name a model and put your own name on it, I think. They had the original idea of dynamic trading as a way to get to a solution. My contribution was to show that the whole idea of replication, the result, was much greater, more general and more robust than they had imagined. I did make a fundamental intellectual contribution, but the dynamic hedging approach certainly was their idea."

When working on this model, did you at any point have the feeling that it would become one of the most well known models in finance and would earn you even the Nobel Prize in Economics?
"Well, I did not expect to win the Nobel Prize. I did know that it was a pretty neat piece of work. I never thought about it earning a Nobel Prize. With any singular prize there are going to be many more people who are fully qualified to get it than do. There is one Nobel Prize a year, sometimes they share it with two or at most three, and you have to be alive to get it. So when you think of the whole world and of new ideas coming along all the time, it is rather naïve to think: 'I am going to get a Nobel Prize for that'. Even if you believe that the work is of such quality and impact that you would qualify, the odds of you receiving it would still be rather low."

If ever you would write an autobiography, where would you lay the focus on and in what perspective would you place the period of working on the Black-Scholes model?
"That is a fair question. It has always been a little bit uncomfortable for me to think about writing an autobiography, but if I were to write one, I think that it

would be focused on the professional part of my life. So it would be a professional autobiography and not a personal one. There is no question that the period from the time I arrived at MIT and became engaged in the subject in an active way until the end of the 70's was an incredibly productive period. That part would be a very big part of my autobiography, because that set my whole mind in the way I would be going forward. I think I grew over the years and I expanded my interest to a broader set of financial research interests, but clearly the most important and intense period were those 10-12 years starting when I was a graduate student at MIT."

Are there any persons you admire? If so, who and why?

"There are many people I admire along various dimensions. I hesitate to respond because I fear I might leave out those who I should mention, not because it is proper, but because they deserve to be. Obviously a very big impact on my professional life was Paul Samuelson. I was his student, he was my mentor, we were co-authors, and we were colleagues and friends for over forty years. My father was a very important influence on my intellectual life. In terms of impact, I would say my father and Paul Samuelson were the most influential."

What advice would you like to give the students in econometrics at Tilburg University?

"There is a very exciting world of quantitative economics and finance out there with a great number of important-to-society and challenging problems to solve. The combination of technology and mindset is having profound changes on how we do things. Having the quantitative skills and techniques is the way the world is going. It is not a question of whether you can be quantitative or not, you have no choice.

The ability to use modeling is not an aside. You build models to implement solutions and then you monitor the models. It is a little bit like flying jet planes. They are going so fast that it is not possible for a human to directly fly the plane, but the human can monitor the computer models and override them when they make mistakes or are incapable of handling the situation. And that offers a good lesson for our field of finance. As a result of the crisis we hear much talk about "good" models and "bad" models. A good model or bad model is not well-defined. What is well-defined as good or bad is a model, the user of the model, and the application of the model; taken as a triplet. And that triplet is what has to be evaluated. A model in the hands of the right person can do great things; in the hands of the wrong persons it can do very poor things. I use the air plane pilot example as a metaphor for understanding this point. A good pilot is not Tom Cruise in Top Gun, a good pilot is a pilot who understands the computer model operating the plane so well that when he is working with the model, he knows when the model is outside the range of things that it can handle. Then a human being takes over because a human being is the best decision-maker we know for unstructured situations. That is more important than having courage or physical prowess. Students need to think about the interfaces of their models. The models are not standalones, they are tools. They have to be crafted to meet the needs of the user and the application. Given the model, you need a properly trained user. You have to specify, it has to say: 'only a person who knows this should use this model'. That is a very important element."

We have read that you will never retire and today you still teach students and do research. What drives you to keep doing this and what is the main point of focus in your current research?

"Why would you ever want to retire from this? It is exciting, it is a global subject and it is high-tech. If you do it well it is not only intellectually challenging, but you can have a positive impact on potentially millions of people for decades.

Right now, I am actually working on the practical implementation of my research on the next-generation retirement funding solution. In fact, I have been to Holland quite a bit to talk about it to the authorities and have implemented it there. One of the things I am also doing, and an important reason I came back to MIT, is working on a new teaching program, Master in Finance, to produce a much larger number of highly trained finance professionals. I want to flood the world, not just in the private sector, but in the public sector as well, with highly trained young people. That is what the answer to solve the financial challenges we currently face is going to take."

'There is a very exciting world out there'

The Origins of the Black-Scholes Formula

PECIAL

The Black-Scholes equation appears in a paper by Fischer Black and Myron Scholes that was published in 1973 in the Journal of Political Economy. Fischer Black has stated in a later publication that he had arrived at the equation already in 1969, but at the time was unable to solve it, even though he tried really hard.

Name:
Hans Schumacher

Position:
Full Professor

He writes: "I stared at the differential equation for many, many months. I made hundreds of silly mistakes that led me down blind alleys. Nothing worked."

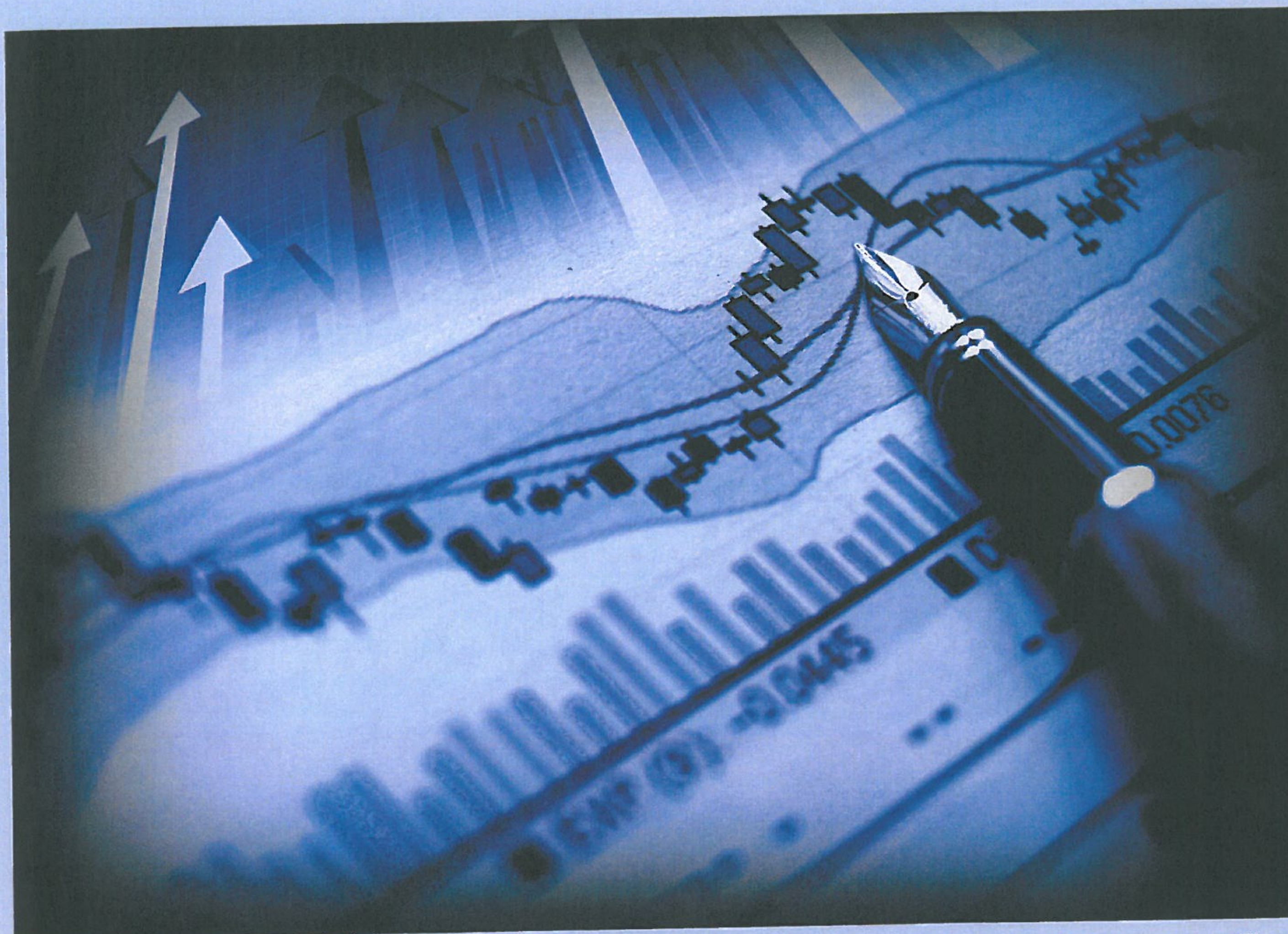
Fischer Black had come into economics from an unusual angle. He entered Harvard University in 1955 as a physics student, but switched to Applied Mathematics for his graduate programme. The PhD thesis that he completed in 1964 was on artificial intelligence, showing the design of a question answering machine. He subsequently joined the consulting firm Arthur D. Little, with the idea of helping businesses to make better use of their computers. It was there that he became interested in portfolio management and started reading the works of people such as Jack Treynor, one of the early proponents of the Capital Asset Pricing Model.

Treynor had published a paper in 1965 in the Harvard Business Review, in which he argued that there should be an adjustment for risk in assessing the performance of portfolio managers, since, due to the presence of a risk premium, more risky portfolios will on average have better returns than less risky portfolios. Fischer Black liked the "cruel truth", as he called it, that higher average return only comes at the expense of higher risk. He tried to apply the idea in several areas that interested him, such as monetary theory, business cycles, and the pricing of options and warrants.

Warrants are financial instruments that are similar to options: they give the right, during a certain period, to buy a given number of units of stock of a certain company at a stated price. The difference is that warrants are issued by the same company that also issues the underlying stock, whereas options are traded on an exchange. For the purpose of pricing, however, this is inessential. During the 1960's warrants were more liquidly traded than options, so that papers discussing the pricing of such instruments were usually stated in terms of warrants rather than options. Among those who were interested in finding option pricing formulas was Paul Samuelson, one of the great minds of the 20th century, who in 1970 became the first American to receive the Nobel Prize in Economics.

Samuelson had done a bit of trading in warrants on a private account already since 1950, without making a lot of money though. Around 1952 he became aware of the work of the French trader and mathematician Louis Bachelier, who had connected the theory of Brownian motion with financial markets in his thesis presented at the Sorbonne in Paris in the year 1900. Even earlier, in 1880, the Danish actuary Thorvald Thiele published a paper on the least-squares method in which the stochastic process appears that we now call the "Wiener process" or "Brownian motion". Bachelier, however, was not aware of this work and developed the theory completely by himself, including the connection to partial differential equations which was to be rediscovered, again independently, in 1905 by none other than Albert Einstein. Options were traded at the Bourse at the time, and Bachelier derived an option pricing formula.

It was not only the option pricing formula that drew Samuelson's attention, but also the mathematical setting that Bachelier had used. Samuelson noted that the Brownian motion process as used by Bachelier (also known as arithmetic Brownian motion) would not be suitable as a model for stock prices, since it may well take negative values. Famously commenting that "a stock might double or halve at commensurable odds", Samuelson proposed a model in which the logarithms of stock prices



follow a Brownian motion process, rather than the prices themselves. Thus appeared the geometric Brownian as a model for stock prices. Nowadays this model is usually referred to as the Black-Scholes model, since it serves as the basis for the Black-Scholes equation and the Black-Scholes formula for option prices, but it would actually be more appropriate to refer to it as the Bachelier-Samuelson model, since it arose as Samuelson's modification of Bachelier's original proposal for the modeling of stock prices. We can then still abbreviate it as the BS model.

The theory of Brownian motion was made mathematically rigorous in the 1930s by Norbert Wiener, and during the 1940s and 1950s the theory was expanded to a great extent by Kiyoshi Itô, who developed a stochastic calculus that could be used, for instance, to formulate stochastic differential equations. Samuelson, not feeling quite confident in the use of the new calculus himself, wrote a paper on the pricing of warrants in 1965 in collaboration with Henry McKean, his colleague from the MIT mathematics department who in the same year published a book on diffusion processes jointly with Itô. Despite the strong mathematical foundation, the pricing formula that Samuelson obtained in this paper was still not satisfactory, since it contained some undetermined parameters. In the 1960s, several other pricing formulas were proposed, however, they all suffered from the same problem.

Samuelson was well aware of the deficits of his formula. Looking for someone who could support him in the further mathematical developments that would be needed, he was happy to notice among the participants in his graduate course in 1967 a student who had just come in from California Institute of Technology as a result of a switch from applied mathematics to economics.

In the spring, Samuelson hired the student, whose name was Robert C. Merton, as his research assistant, and in the summer he proposed that they would write a joint paper on the pricing of options. The paper appeared in 1969; it eliminated the undetermined parameters of Samuelson's earlier paper, but only at the expense of invoking an explicit description of the preferences of agents by means of utility functions. In October 1968, when Samuelson was announced to deliver the main lecture at the inaugural session of the MIT-Harvard Joint Seminar in Mathematical Economics, he surprised the luminaries in his audience by instead giving the floor to his 24-year-old PhD student, in order to present their joint paper on option pricing. Merton later recalled that this experience at once cured him from any trepidation for audiences.

Myron Scholes arrived in the Boston area in the fall of 1968 as a starting assistant professor at MIT's Sloan School of Management, having just completed a PhD at the University of Chicago under the direction of Merton Miller. One of the people he made contact with in his new environment was Fischer Black, who was a regular visitor at Franco Modigliani's Tuesday night finance seminars at MIT, and whose office at Arthur D. Little was located close to the MIT campus. When Wells Fargo, one of the most innovative banks at the time, offered Scholes a consulting position, he suggested that they would hire Fischer Black as well. As a result Black and Scholes came to meet regularly, be it no longer at Arthur D. Little but rather at Black's own consulting practice which he had started after quitting his job at ADL.

The two men talked about many things, but not about options at first. Then, sometime in 1969, Black showed the equation he had derived to Scholes, and discussed with him the remarkable fact that the expected return on the underlying stock plays no role in it. From this observation, they concluded that candidate solutions to the equation might be found from simplified versions of the option pricing formulas that were already around in the literature. And indeed, working from a formula that was developed by a Yale University graduate student, they arrived at the solution. They had found an option pricing formula that, unlike its competitors, was stated directly in terms of observable quantities.

Fischer Black had arrived at his option pricing equation through an application of CAPM. When Bob Merton came to know about the equation, following a presentation by Scholes at the second Wells Fargo Conference on Capital Market Theory in July 1970, he was skeptical. He could not believe that a static theory like CAPM could be reasonably combined with a theory of continuous or near-continuous trading. Thinking about it some more, he found a different argument leading to the same equation. On a Saturday afternoon in August, he made a phone call to Scholes and said: "You are right."

As they say: the rest is history. Black and Scholes wrote their paper on the option pricing formula and submitted it to the Journal of Political Economy where it was promptly rejected, without even being sent out for review. Subsequently, they sent their paper to the Review of Economics and Statistics, only to have it returned in the same way. At that point, Scholes' former PhD advisor Merton Miller and his colleague Eugene Fama stepped in: they convinced the editors of JPE that the paper might be worthwhile after all. The paper was accepted subject to revision in August 1971, and it finally appeared in 1973, as it happened one month after the Chicago Board Options Exchange had opened for business. Soon, the Wall Street Journal would carry advertisements for calculators with the Black-Scholes formula built in.

The main argument presented for the Black-Scholes equation in the 1973 paper is the one that was provided by Merton. Black's original argument is given as an "alternative derivation". Merton provides yet another derivation in a paper published in 1977, which is only for the better, since the argument as used in the 1973 paper would be considered rather dubious by current standards. Major steps towards the completion of the theory were taken by Michael Harrison together with David Kreps in 1979 and together with Stanley Pliska in another paper published in 1981. In these papers one finds the notions of "self-financing strategy" and "equivalent martingale measure" that are lacking from the original option pricing papers, and that are essential for a full development of the theory even though Harrison and Kreps themselves refer to the EMM as a "somewhat abstruse concept". Other researchers have expanded the theory further, both strengthening its foundations and extending widely its domain of applications.

Fischer Black died of cancer in 1995. Myron Scholes and Robert Merton received the Nobel Prize in Economics in 1997. These three men have been pivotal in the development of a theory that has fundamentally transformed the world of finance.

References

Capital Ideas, by Peter L. Bernstein (The Free Press, New York, 1992)

Fischer Black and the Revolutionary Idea of Finance, by Perry Mehrling (Wiley, Hoboken, NJ, 2005).