

$$D = VN(-d_+) + Fe^{-r\tau}N(d_-)$$

Harvard's Financial Scientist

Robert Merton's option-based model for valuing corporate debt, one of the insights for which he won a Nobel prize, is receiving renewed interest for valuing credit derivatives.

By Peter Carr

◆ ROBERT MERTON, WHO'S best known for his academic work in developing option-pricing models, published his first scholarly paper on a topic far removed from finance: *Gulliver's Travels*.

While an undergraduate at Columbia University in New York, he wrote a piece for a literature class that analyzed the physical impossibility of a flying island as described in Jonathan Swift's classic work. Merton, who majored in engineering mathematics at Columbia, says his English instructor proposed that they publish the paper as co-authors. "I asked myself, 'What did he do?'" Merton says. He submitted "The 'Motionless' Motion of Swift's Flying Island" by himself to the *Journal of the History of Ideas*, which published it in 1966. Merton got a D in the class.

Merton, now a professor of finance at Harvard Business School in Boston, won the Nobel Memorial Prize in Economic Sciences in 1997 along with Myron Scholes for their development of the option-pricing model that many quants refer to as Black-Merton-Scholes.

Merton played an instrumental role in discovering the Black-Scholes model, supplying the crucial arbitrage argument that underpins its logic. In January 1973, Merton was the first to publish the differential equation that became known as the Black-Scholes equation in a paper called "Continuous-Time Speculative Processes." Black and Scholes published their full article on the model later that year.

At about the same time, he developed the Merton model, a method of valuing a corporate bond based on default probability using a form of the Black-Scholes equation. The high-profile defaults of companies such as Enron Corp. and WorldCom Inc. and the rapid growth in the credit derivatives market have

prompted a boom in interest in the Merton model among credit analysts.

In addition to his teaching responsibilities at Harvard, Merton is now chief science officer at Integrated Finance Ltd., a New York-based investment bank he co-founded in December 2002. The firm closed its IFL Continuum Fund in June, according to Beth Burrus, a managing director of Integrated Finance. The fund, which concentrated on fixed-income securities, had collected \$30 million since its start in March and was managed by Peter Hancock. Integrated Finance also has an \$80 million emerging-market hedge fund run by José Luis Daza.

Merton, 62, received his pre-doctorate training in the hard sciences. He earned a bachelor's degree at Columbia and then a master's degree in applied mathematics at California Institute of Technology in Pasadena.

Merton credits his time in the master's program at Caltech with helping him make the transition from passive learning to an active, self-directed education. "They would just throw you in a lab, and you would play around," he says. He also spent his early-morning hours before class trading convertible bonds and warrants, which give the holder the right to buy the underlying stock, in the over-the-counter market. He says he became increasingly interested in finance and impressed by the idea, which was common at the time, that macroeconomic theory could solve many of the world's economic problems. He decided to apply to doctoral programs in economics after he completed his master's degree.

WHEN HE APPLIED to Ph.D. programs in economics at seven or eight schools in 1967, every one turned him down

Robert Merton

Merton model for valuing corporate debt

Current job: John and Natty McArthur University Professor at Harvard Business School in Boston. Also chief science officer and co-founder of Integrated Finance in New York.

Background: Earned a bachelor's degree in engineering mathematics from Columbia University in 1966. Received a master's degree in applied mathematics from California Institute of Technology in 1967. Earned a Ph.D. in economics from Massachusetts Institute of Technology in 1970. Was professor of finance at various levels at MIT's Sloan School of Management from 1970 to '88, when he moved to Harvard Business School.

Personal: Age 62. Born in New York. Has two sons and a daughter and four grandchildren.

flat—except for Massachusetts Institute of Technology in Cambridge. At the time, it was unusual for scientists to seek to study economics or finance, a situation that Merton would soon change. While at MIT, Merton caught the eye of Paul Samuelson, who won the Nobel prize in economics in 1970. Samuelson, a professor of economics at MIT, invited Merton to share his office, and they co-authored an article on warrant pricing. "Samuelson is the last truly general economist," says Merton, who attended a 90th-birthday celebration for Samuelson earlier this year.

While working with Samuelson, Merton introduced stochastic integrals to finance. In the late 1960s, stochastic calculus was an obscure field known only to a subset of mathematicians and engineers. Stochastic calculus involves

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random functions whose outcome can't be predicted. In the 1940s and '50s, Japanese mathematician Kiyoshi Itô developed a way to define a stochastic integral. Merton applied this method to academic research in finance, using it to describe the gains from dynamic trading of securities. These integrals of stochastic functions are now an essential tool in quantitative finance.

When it came time to graduate with his Ph.D., Merton didn't have to go far. He joined the finance department of MIT's Sloan School of Management in 1970.

Armed with techniques borrowed from stochastic calculus and dynamic programming, Merton solved the problem of how an investor's wealth should be allocated in an intertemporal setting—one that involves more than one time period. Dynamic programming is a method of theoretical optimization that solves problems by moving backward through time.

TO SIMPLIFY THE problem, Merton assumed that any wealth not consumed would be allocated between a riskless asset, such as a U.S. Treasury note, and a single risky asset. Even after this simplification, he had to solve a daunting nonlinear partial differential equation that arises out of an aspect of dynamic programming. Merton nonetheless guessed the correct solution and then proceeded to show that it's possible to economically justify the assumption that investors split their wealth between just two funds.

In particular, by formulating the capital-asset-pricing model in continuous time, Merton showed that the single risky asset could be taken to be the market portfolio, the unique portfolio whose weights are proportional to market value. He also showed that randomness in the statistical inputs would cause demand for additional risky portfolios, generalizing the standard two-fund separation results to an arbitrary number of funds.

In 1970, Merton published a working paper at MIT called "A Dynamic General Equilibrium Model of the Asset Market and Its Application to the Pricing of the Capital Structure of the Firm." In it, he showed how to apply derivatives pricing theory to quantify the possibility of default

by the issuer of a corporate bond. Merton says the idea behind the paper came about as a result of his work on the option valuation theory that eventually led to the Black-Merton-Scholes formula. He developed the corporate bond default theory into a model for analyzing corporate debt and published it in the 1974 proceedings issue of the *Journal of Finance* as "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." This theory is now known as the Merton model for valuing corporate debt.

Assuming for simplicity that the bond has no coupons and that the issuer has no other debt, Merton showed that the only possible default time would be at the debt's maturity. Given any random process for determining the value of the firm's assets, Merton assumed that default would occur at maturity if the value of those assets couldn't support the promised repayment of principal. In the event of default, bondholders would seize the firm's assets and liquidate them, cutting their loss to the difference between the promised payment and the liquidation proceeds.

Of course, in the event default doesn't occur, bondholders suffer no loss. Consequently, adding the possibility of default to an otherwise riskless payment reduces its realization by the value of a European put option written on the firm's assets. A European put gives the holder the right to sell the underlying assets on the day of expiration—which, in this case, is the date that the bond matures. The option's strike price is the promised payment from the bond.

Since you can't directly observe the price of the put from options markets, Merton needed a model for the put value. He assumed the firm's value follows a continuous process with constant volatility. That allowed him to use the Black-Merton-Scholes formula for the value of a put option. The net result for the price of a zero-coupon corporate bond is:

$$D = VN(-d_+) + Fe^{-r\tau}N(d_-),$$

$$N(d) \equiv \int_{-\infty}^d \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz,$$

$$d_{\pm} = \frac{\ln(V/F) + (r \pm \sigma_v^2/2)\tau}{\sigma_v\sqrt{\tau}}.$$

Here:

- D is the unknown debt value.
- V is the assumed known initial asset value.
- F is the face value of the debt, the fixed amount that the bond promises to pay its holders at maturity.
- r is the default-free interest rate for both borrowing and lending, assumed constant over the put's lifetime τ .
- $N(d)$ denotes the normal distribution function, the probability that a standardized Gaussian random variable realizes below d .
- d_+ and d_- are alternative measures of the distance to default, indicating by how many standard deviations $\sigma_v\sqrt{\tau}$ the concavity corrected return $\ln(V_T/V)$ exceeds the debt-to-asset ratio $\ln(Fe^{-r\tau}/V)$.

The key inputs to the formula are the firm's initial asset value V and the firm's asset volatility σ_v , a positive constant that measures the variability in asset returns over time. In principle, these two inputs can be determined from estimates of the initial stock price and stock volatility. Figure 1 shows the payoff of corporate debt according to the model.

"The paper didn't exactly take the world by storm," Merton says. One possible explanation for the lukewarm reception is that Merton's modeling of default as a put option had already been mentioned in the 1973 Black-Scholes paper. In the first half of the Black-Scholes study, the two authors assumed the equity volatility is constant. In the second half of the paper, they inconsistently assumed that the asset volatility is constant.

As Merton showed in his paper, constant asset volatility produces nonconstant equity volatility. The only dynamics for firm value that would produce constant equity volatility require that firm value arise as the sum of a constant volatility process and a riskless asset worth face value at expiration. Since the corporate debt is then completely collateralized by the riskless asset, Merton's default put becomes worthless.

There are several other possible reasons why Merton's corporate-bond-pricing

results weren't immediately adopted and celebrated as they are today. Perhaps it was the later empirical work that Merton's students and others published in a 1984 paper in the *Journal of Finance* that showed that the model overvalued short-term corporate debt. It's now known that a large component of a corporate bond's value comes from measures of illiquidity as opposed to default. Perhaps academics recognized that jumps in asset value are an important reality, especially when pricing short-term debt.

IN HINDSIGHT, WE see that Merton's suggestion of a constant-volatility, continuous process for firm value was only a tractable suggestion. Merton and others eventually applied his own later work on modeling jump processes to better describe the firm's asset dynamics, which explained away some of the Merton model's short-term mispricing.

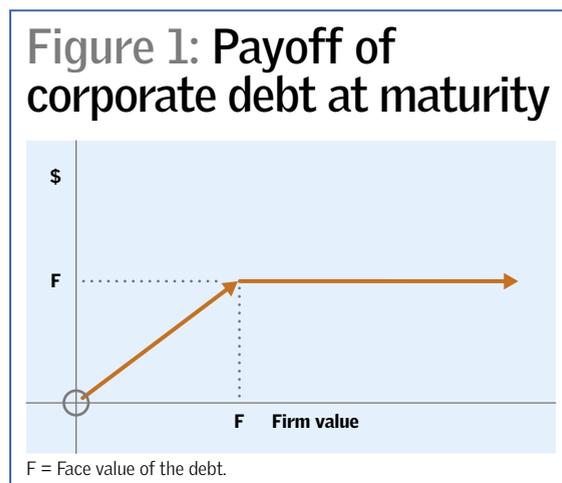
Perhaps the main reason Merton's model wasn't initially adopted is that his assumptions of no coupons and no other debt were just too simplistic. Building on Merton's model, a San Francisco-based company called KMV, founded in 1989, amended Merton's assumption that default could occur only at maturity. Instead, it assumed that effective default occurs the first time firm value falls below some threshold set below the initial firm value. KMV continued to refer to this altered model as the Merton model, a moniker that remains in use to this day. The KMV adaptation of Merton's original model has become an industry standard. Moody's Corp., the second-largest credit rating firm, acquired KMV for \$210 million in 2002.

Merton attributes the more-recent adoption of his corporate debt model to the increasing number of defaults and other credit problems that corporate issuers have experienced in the past 10 years or so.

He cites Stamford, Connecticut-based copier and printer maker Xerox Corp. as

an example. Xerox's credit ratings fell into the junk category in 2000. None of the popular credit models at the time predicted this event, Merton says.

The Merton model did signal an upcoming drop in the value of the company's debt. That's because Xerox's stock price fell sharply before the downgrades, lowering the firm value used in the Merton model and the theoretical



Source: Bloomberg

price of its debt. "The equity markets can give you important information about a credit," Merton says. Xerox shares reached their highest price in 2000—\$29.75—on March 21 before falling to \$4.63 at the end of the year.

MOODY'S INVESTORS SERVICE downgraded the company's debt to junk status on Dec. 1, 2000. Merton says that this type of credit event evaded other models for valuing corporate debt, creating a need for different models. "It's not about smartness; it's about need," he says.

The rapid increase in the size of the credit derivatives market has fueled demand for any model that has the ability to predict default. Analysts refer to models such as those developed by Merton and KMV as structural models because they attempt to explain why default occurs. In structural models, the values of credit derivatives depend on some notion of distance to default. Fundamentally, credit derivatives can

only be valued by making some assumption about the link between the statistical default frequency, which is hard to observe, and the values of risk-neutral default probabilities. Many credit derivative analysts refer to the Merton model as the basis for their own formulas for predicting default.

Today, Merton splits his time between teaching MBA and doctoral seminars at Harvard Business School and developing new ideas for Integrated Finance. At that company, Merton uses the financial engineering techniques he invented to develop new types of retirement savings plans. He favors pension plans that are backed by a combination of the superior returns provided by equities and the financial guarantees offered by different combinations of fixed-income securities and derivatives.

One major difference between quantitative finance today and the field 30 or 35 years ago, according to Merton, is that rapidly increasing computing power has taken away much of the importance of intuition. Merton recalls his graduate students' needing to spend considerable time looking up possible solutions to functions in books he kept in his office.

One book, which they called "big red" based on its size and color, was the American Mathematical Society's *Handbook of Mathematical Functions*. Another, "medium yellow," contained solutions to 500 partial differential equations.

Today, Merton's students can use various software tools to quickly solve a wide range of mathematical problems, eliminating the need for "big red" and "medium yellow." The relative ease of solving mathematical equations today may have inadvertently contributed to a reduced understanding of the underlying financial concepts. Merton's own intuition about finance has served him well. ♦

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